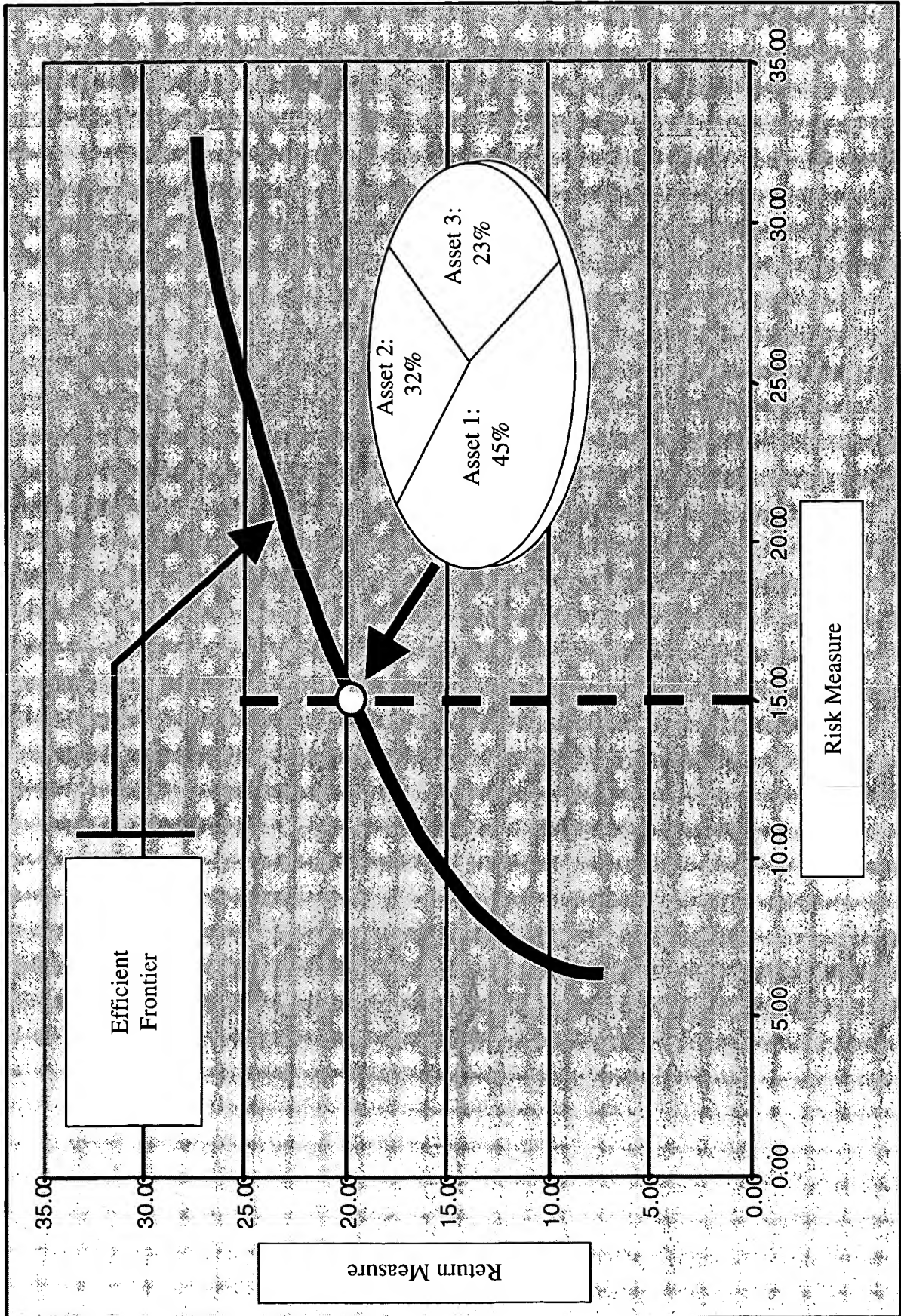
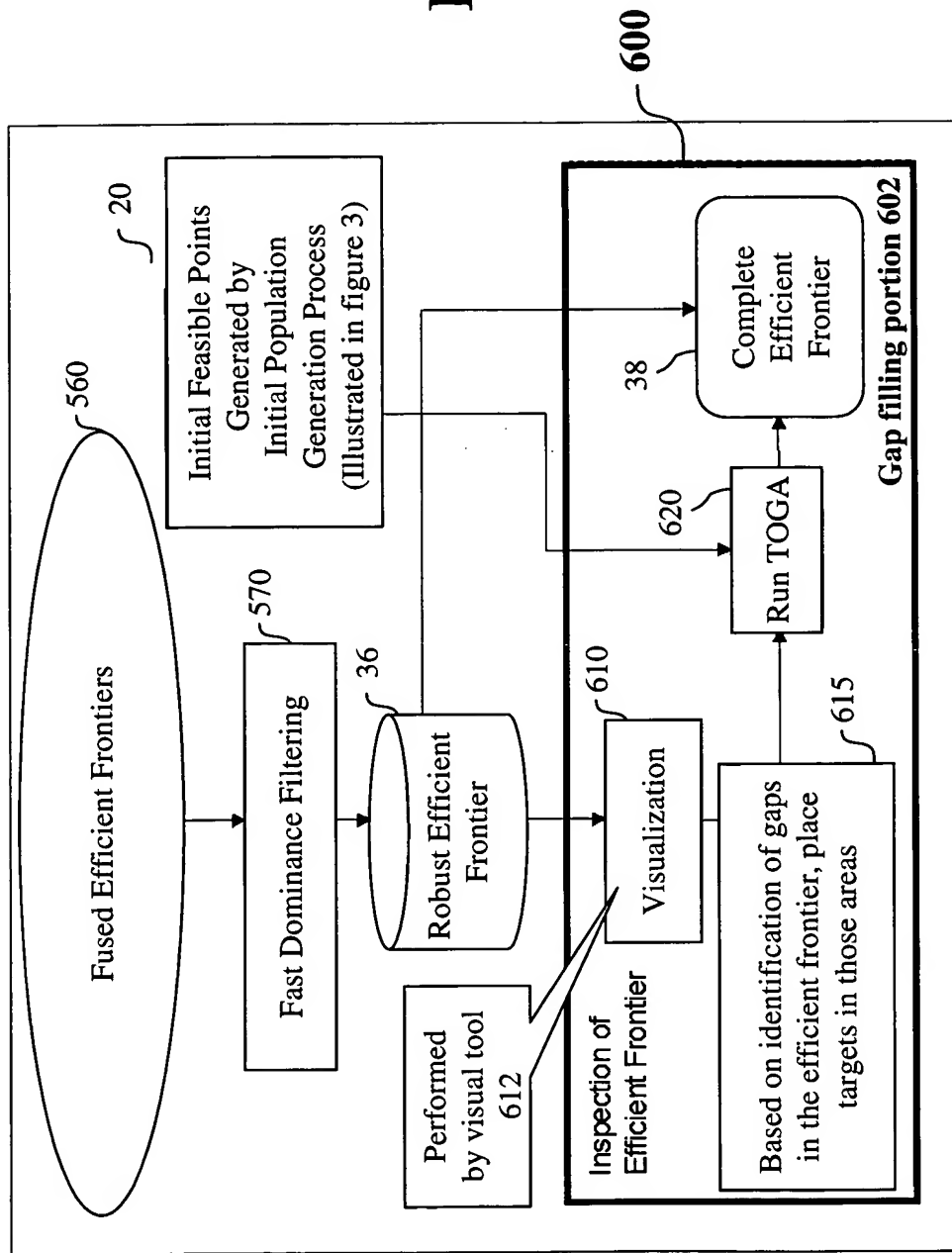


Fig. 1

BACKGROUND ART



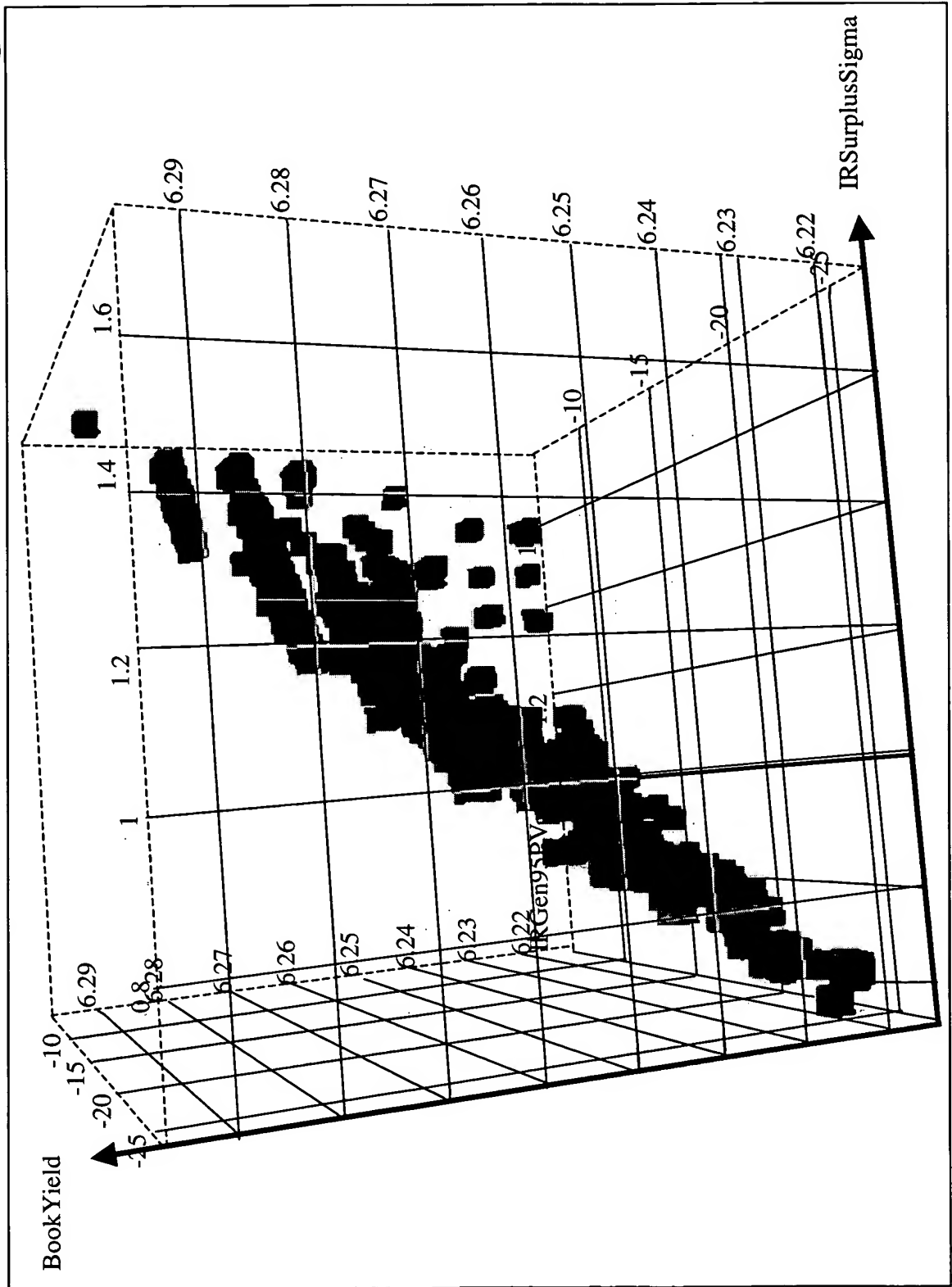


**Fig. 12**

**Process to interactively fill any gaps in the identified efficient frontier**

Fig. 13

Efficient Frontier in a 3D View



EXAMPLE OF PARALLEL COORDINATE PLOT

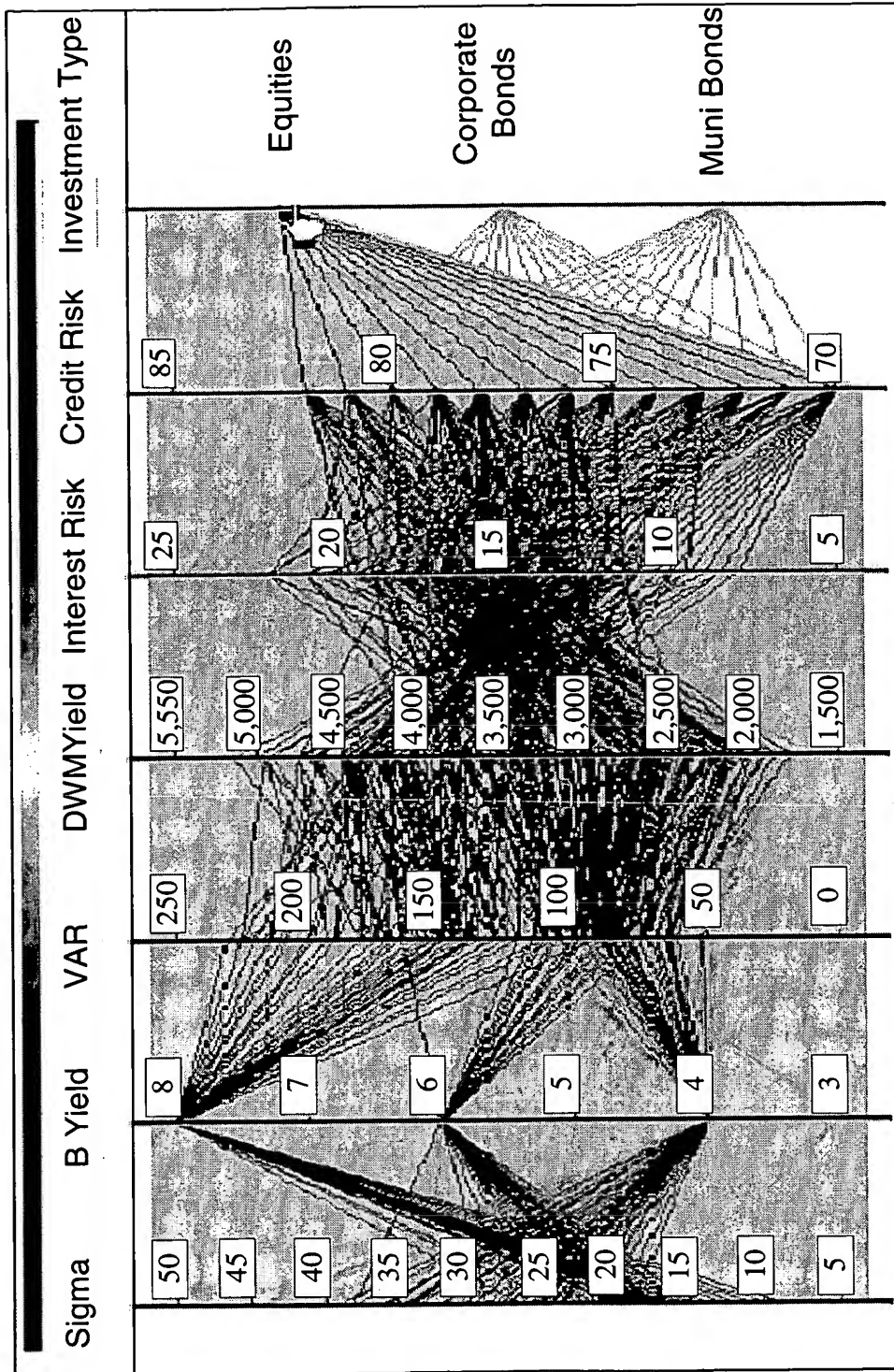
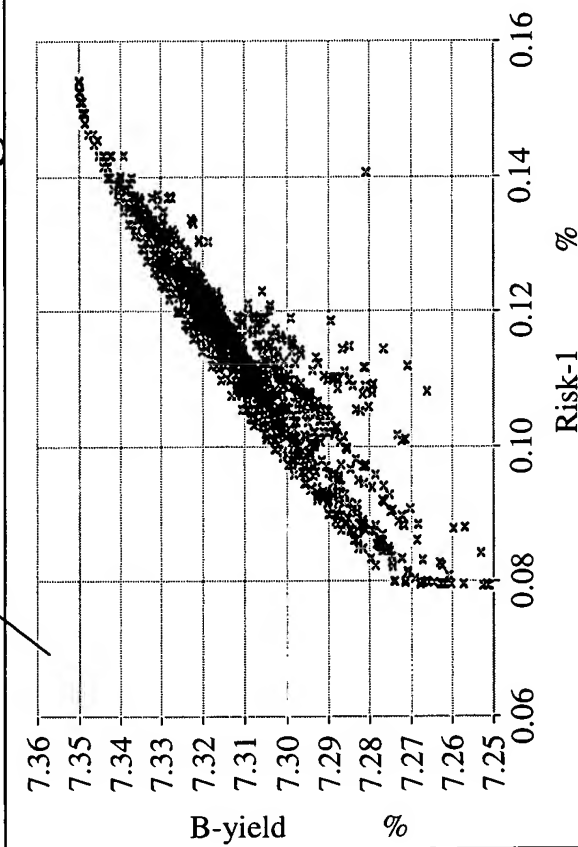


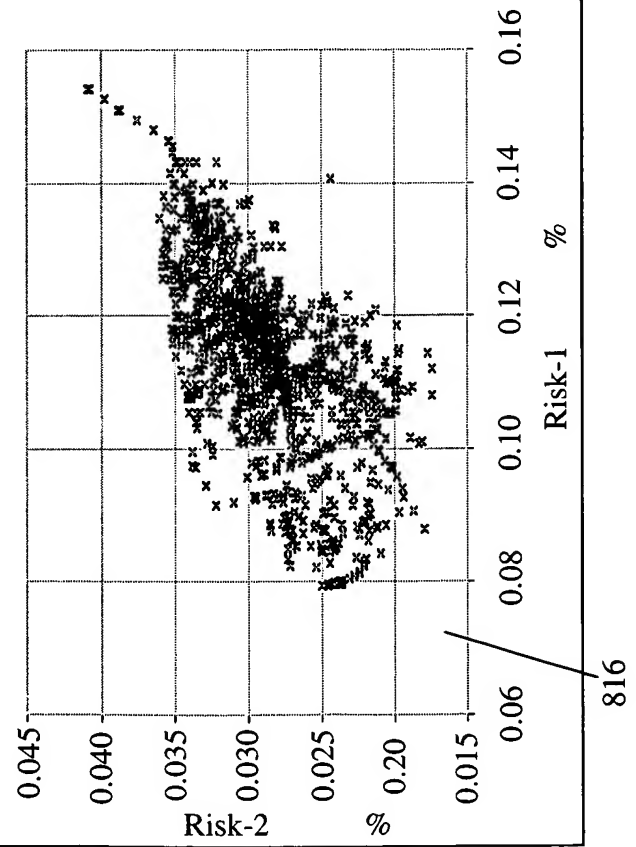
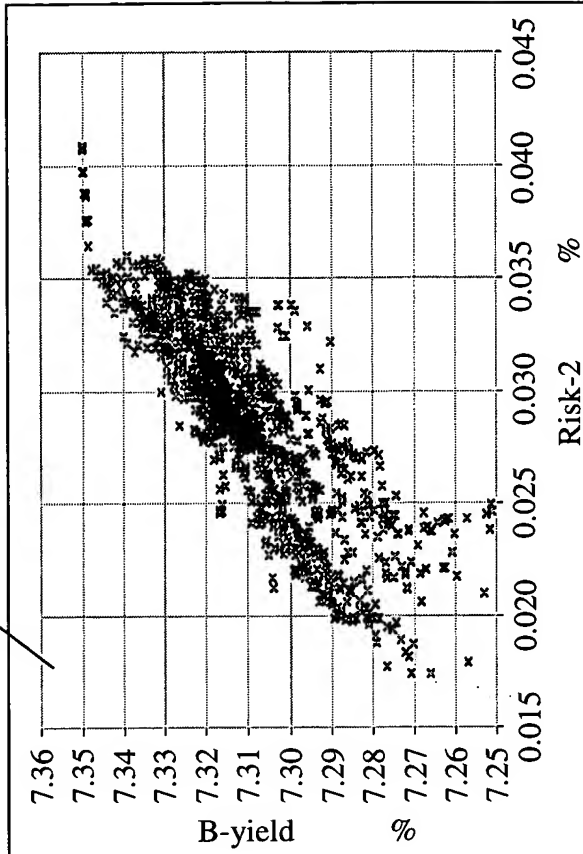
Fig. 14

Fig. 15

812

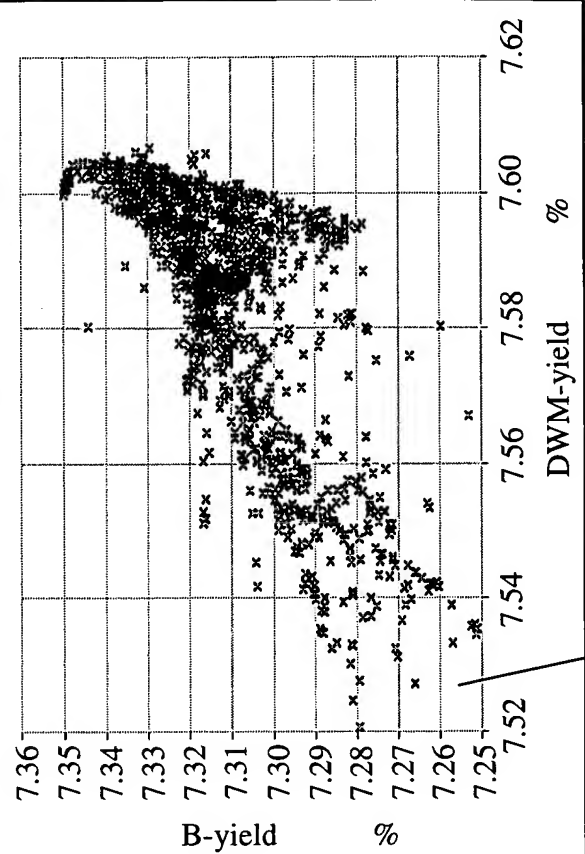


814



816

818



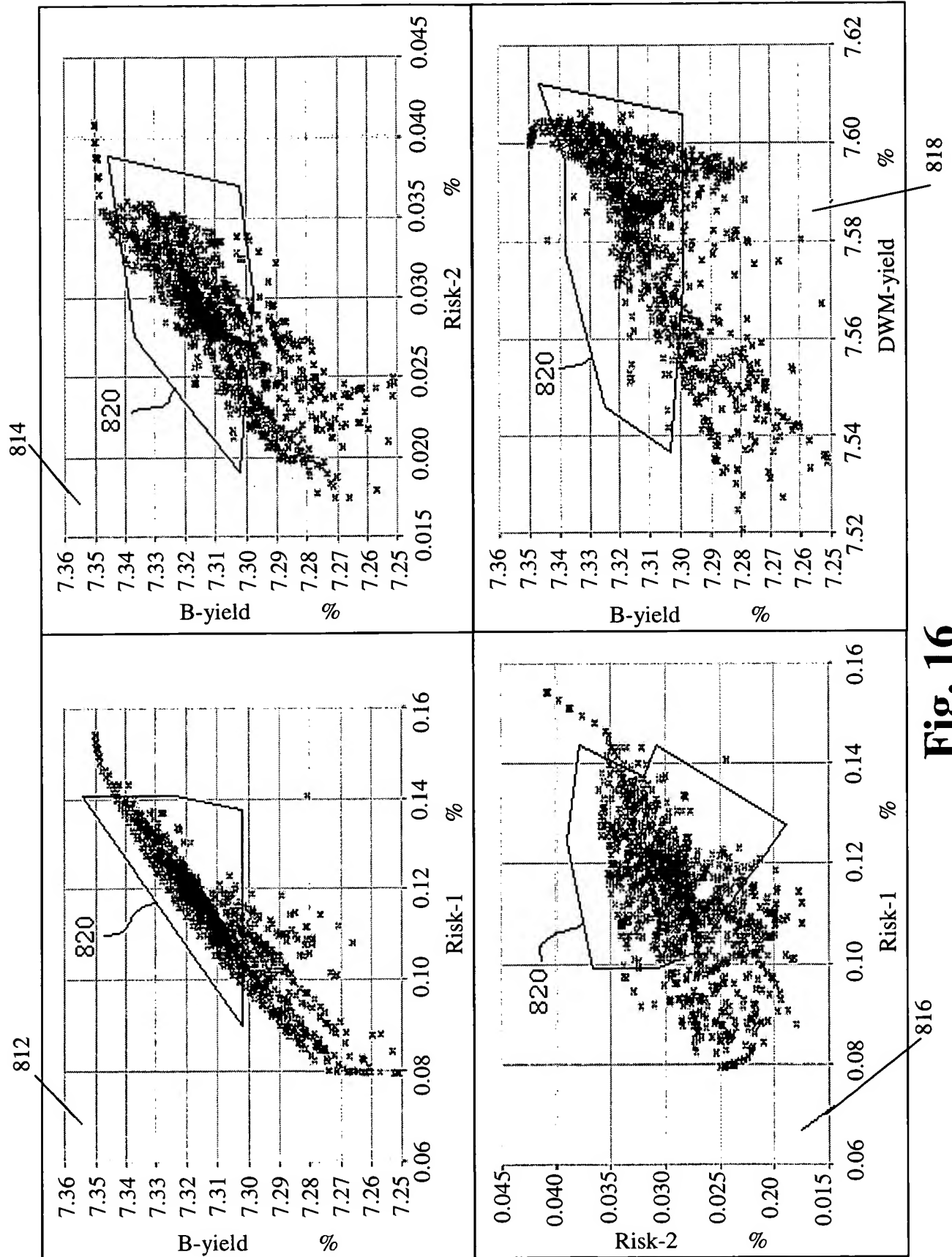


Fig. 16

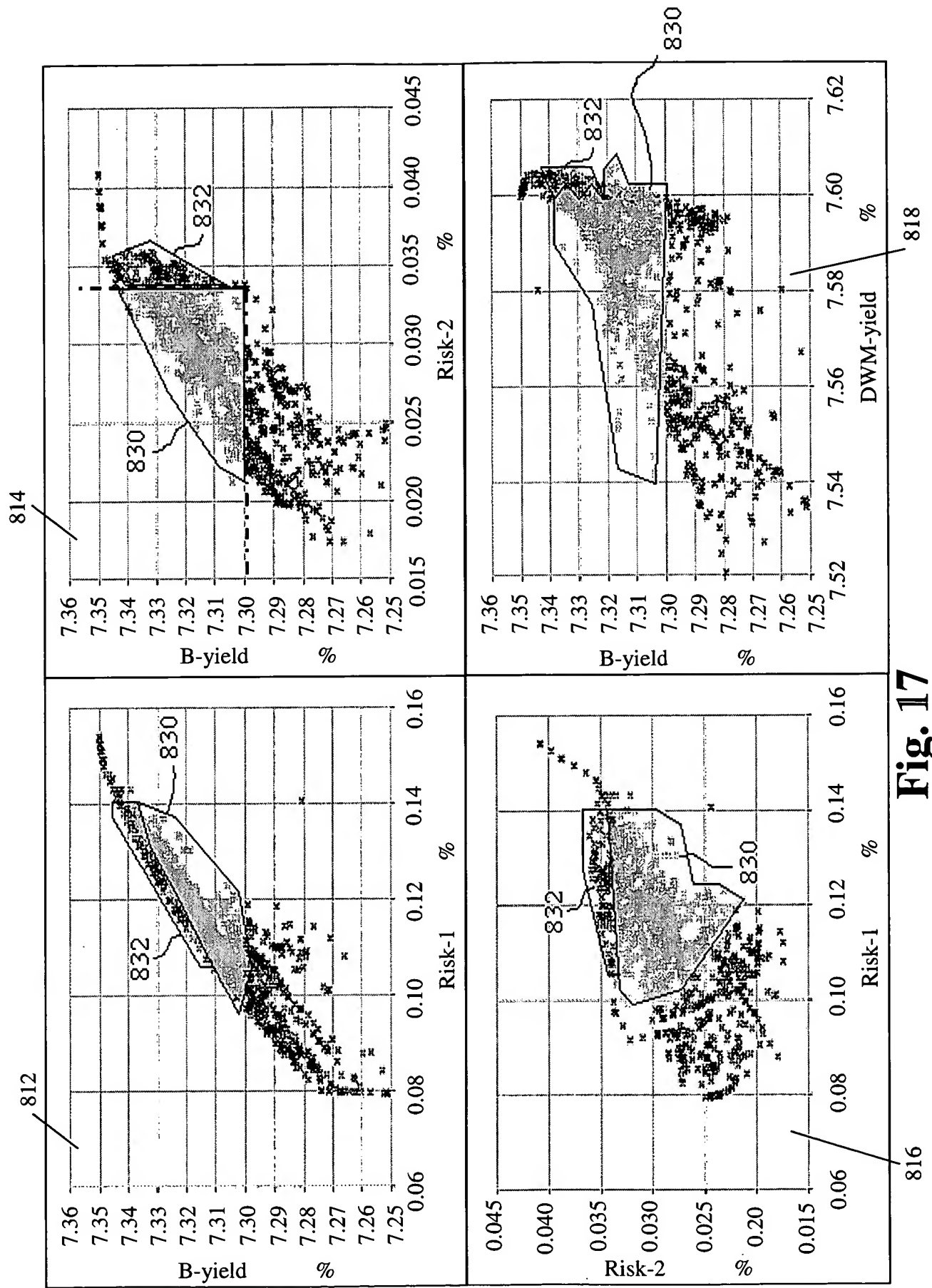


Fig. 17

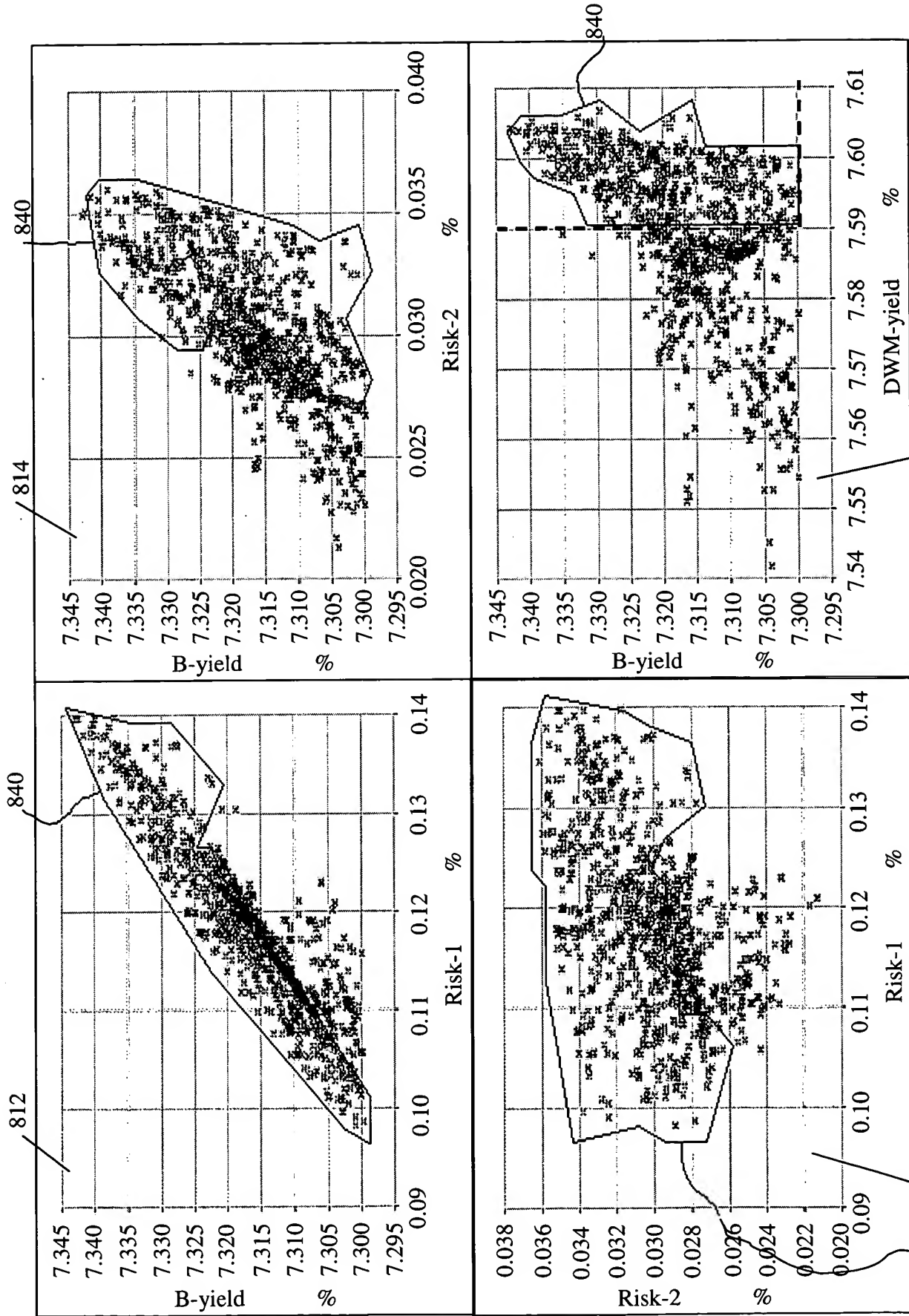
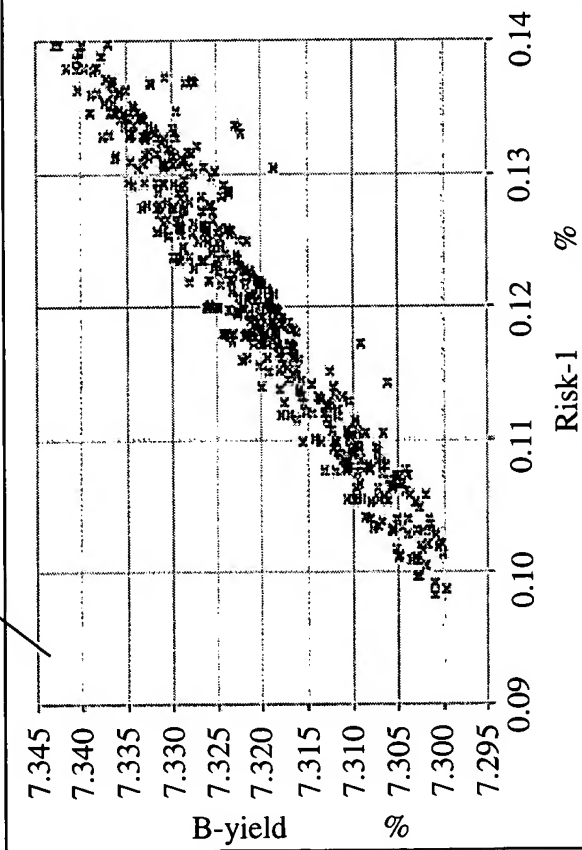
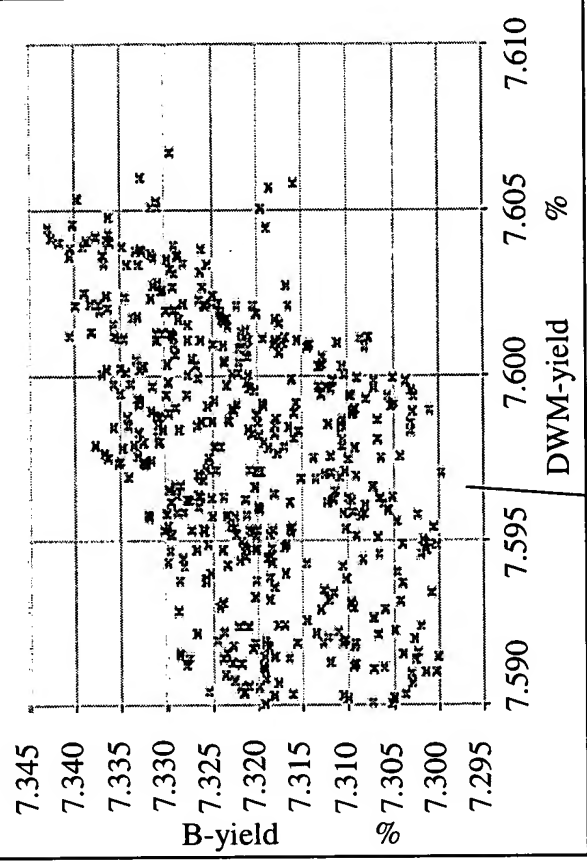
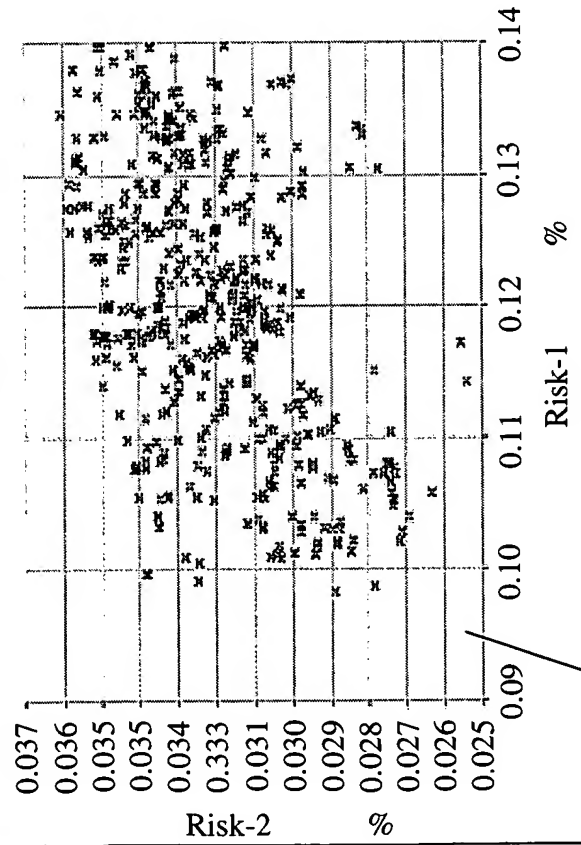
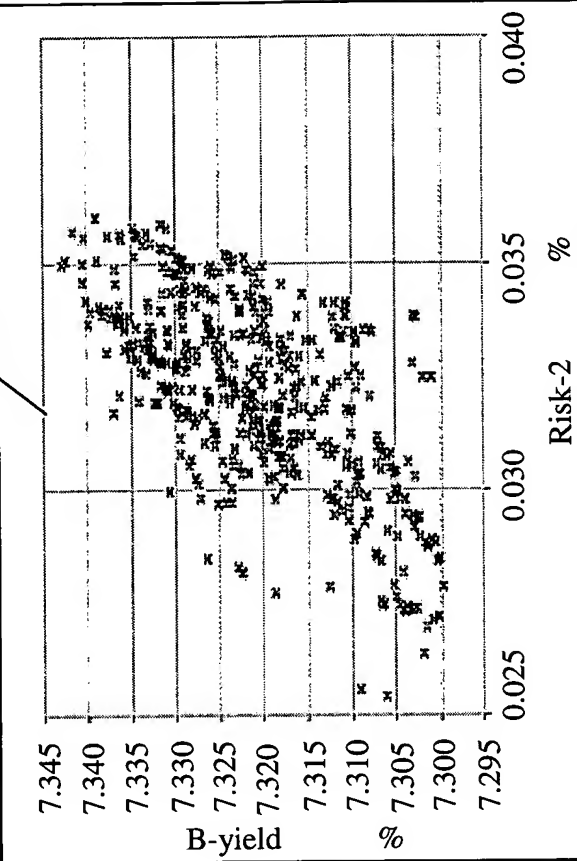


Fig. 18

812



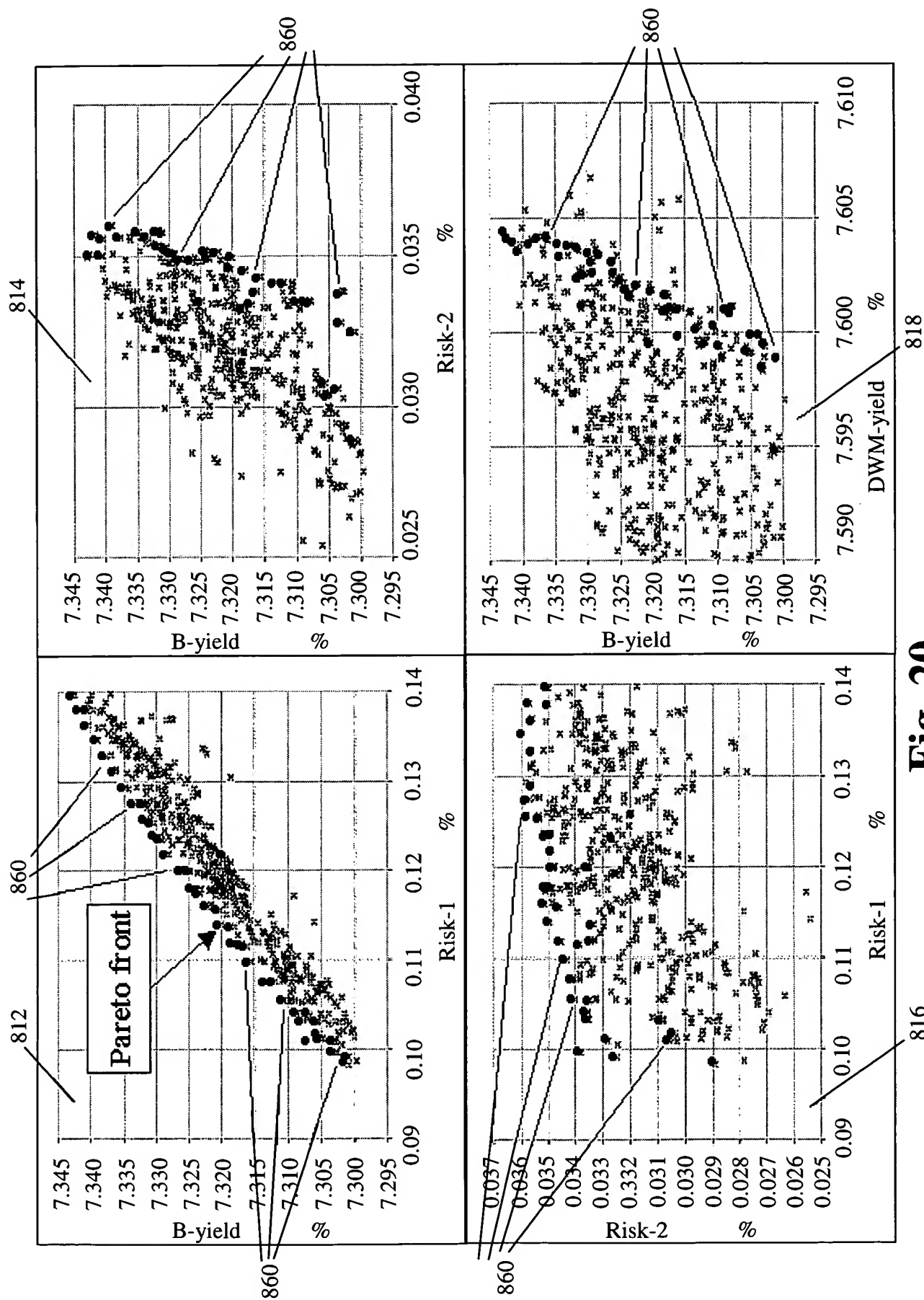
814



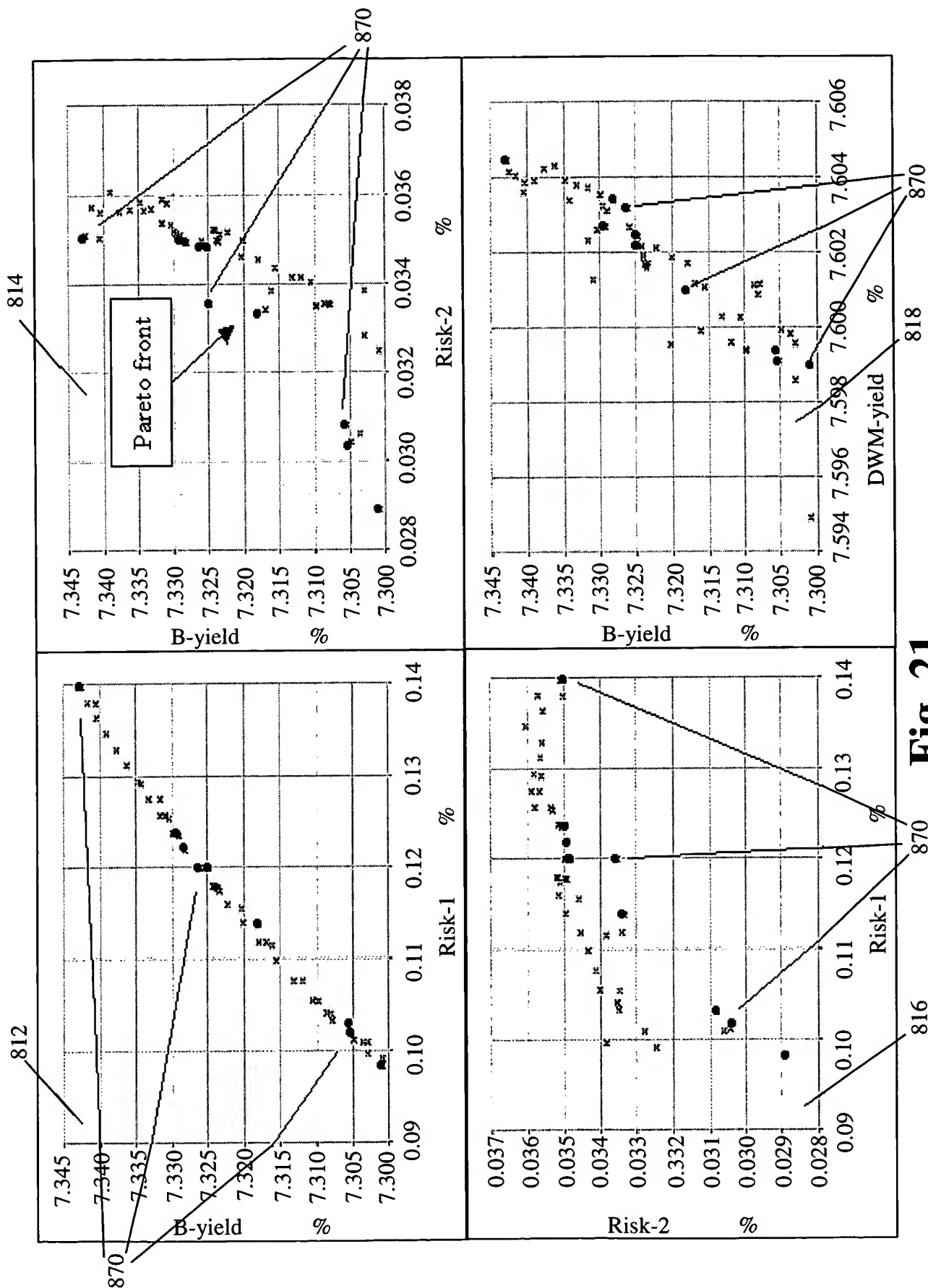
816

Fig. 19

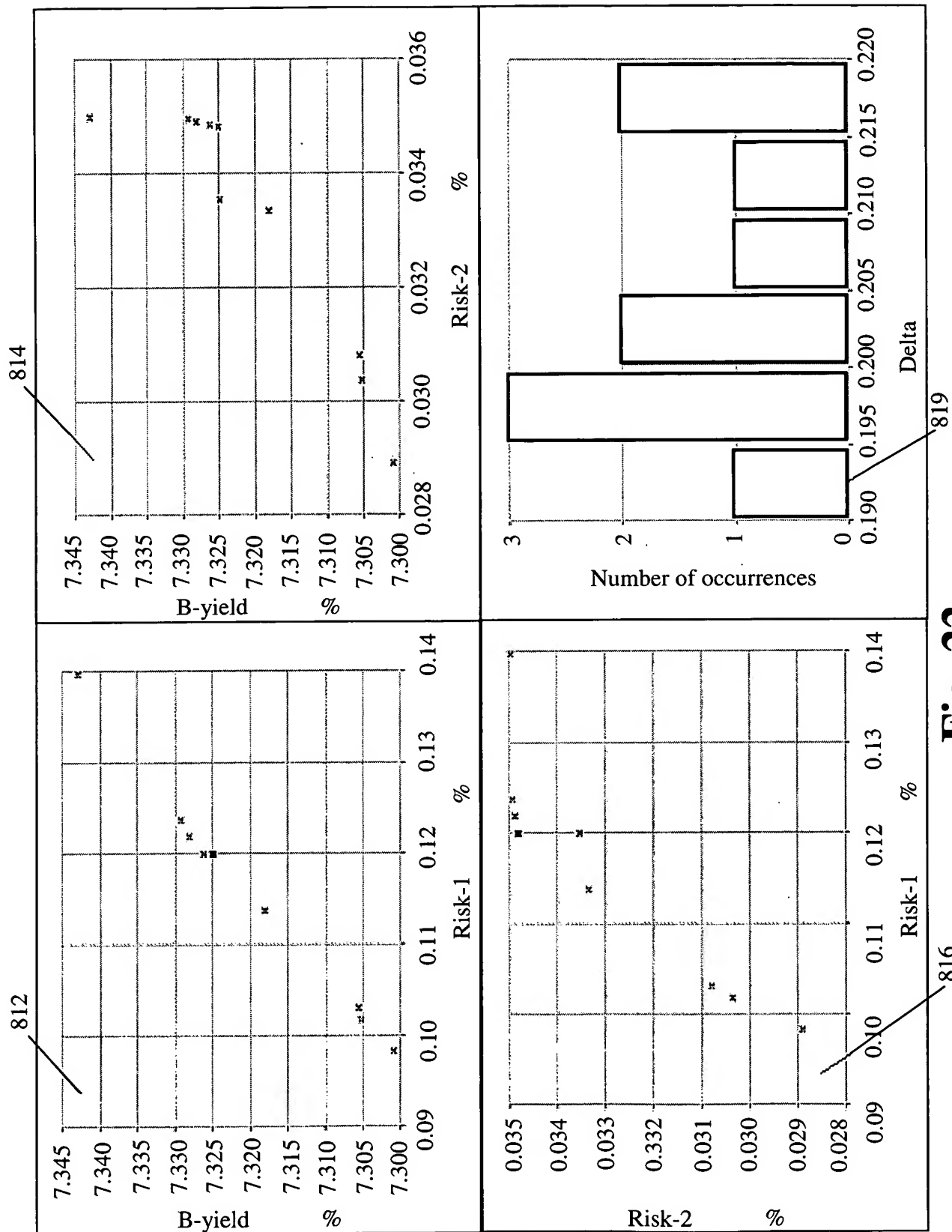
818



**Fig. 20**



**Fig. 21**



**Fig. 22**

# Feasible Regions for Optimization

Figure 33

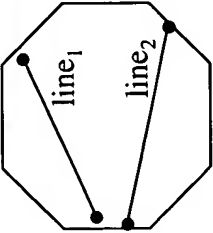
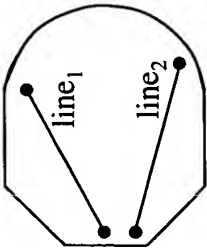
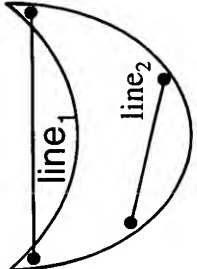
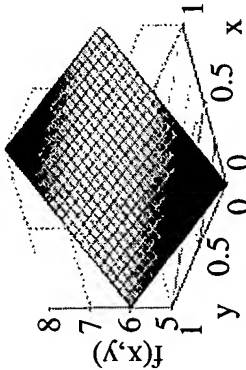
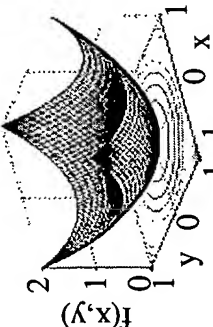
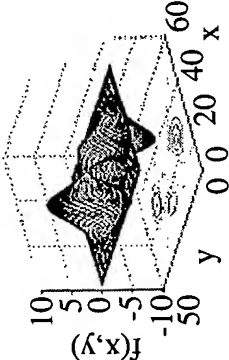
| Graphic Visual                                                                                                             | Word Description                                                                                                                                                                                                                | Example Equation                                                                                                                                                                                                                                               | GEAM                                                                                                                                       |
|----------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| <div>Linear Convex Space</div>          | <ul style="list-style-type: none"> <li>For any two points in the space, the line connecting the two points is <b>always</b> contained in the same space</li> <li>Space is defined using linear equations</li> </ul>             | $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \vdots & \vdots \\ a_{81} & a_{82} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_8 \end{bmatrix}$ <p>Set of linear equations</p>                    | <ul style="list-style-type: none"> <li>Market value weighted yield formulation</li> <li>Duration weighted yield formulation</li> </ul>     |
| <div>Nonlinear Convex Space</div>      | <ul style="list-style-type: none"> <li>For any two points in the space, the line connecting the two points is <b>always</b> contained in the same space</li> <li>Space is defined using some nonlinear equations</li> </ul>     | $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \vdots & \vdots \\ a_{51} & a_{52} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_5 \end{bmatrix}$ <p>Nonlinear equation</p> $x^2 + y^2 \leq \alpha$ | <ul style="list-style-type: none"> <li>Interest rate sigma formulation</li> </ul>                                                          |
| <div>Nonlinear Nonconvex Space</div>  | <ul style="list-style-type: none"> <li>For any two points in the space, the line connecting the two points is <b>not</b> always contained in the same space</li> <li>Space is defined using some nonlinear equations</li> </ul> | $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} x^2 \\ x \\ y \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ <p>Set of nonlinear equations</p>                                               | <ul style="list-style-type: none"> <li>Interest rate sigma and VAR formulation</li> <li>VAR is a nonlinear nonconvex constraint</li> </ul> |

Figure 34

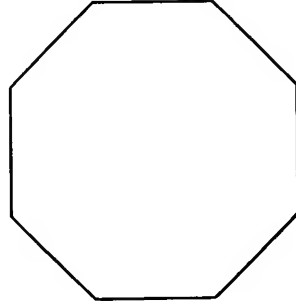
## Objective Functions

| Graphic Visual                                                                                                                   | Word Description                                                                                                                                                                                                            | Example Equation                                          | GEAM                                                                                                               |
|----------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------|
| <p><b>Linear Function</b></p>                 | <ul style="list-style-type: none"> <li>• Function is defined using linear equations</li> <li>• Straightforward math relationship</li> <li>• Easy to optimize</li> </ul>                                                     | $f(x, y) = 2x + y + 5$                                    | <ul style="list-style-type: none"> <li>• Market value weighted yield</li> <li>• Duration weighted yield</li> </ul> |
| <p><b>Nonlinear Convex Function</b></p>      | <ul style="list-style-type: none"> <li>• Function is defined using a nonlinear equation</li> <li>• Functional gradients lead to single optimum</li> <li>• Harder to optimize</li> </ul>                                     | $f(x, y) = x^2 + y^2$                                     | <ul style="list-style-type: none"> <li>• Interest rate sigma</li> </ul>                                            |
| <p><b>Nonlinear Nonconvex Function</b></p>  | <ul style="list-style-type: none"> <li>• Function is defined using complex nonlinear equations</li> <li>• Multiple local optima</li> <li>• Functional gradients are inefficient</li> <li>• Very hard to optimize</li> </ul> | $f(x, y) = g_1(x, y) + g_2(x, y) + g_3(x, y) + g_4(x, y)$ | <ul style="list-style-type: none"> <li>• Interest rate sigma and VAR</li> </ul>                                    |

### *Evolutionary Search Augmented with Domain Knowledge*

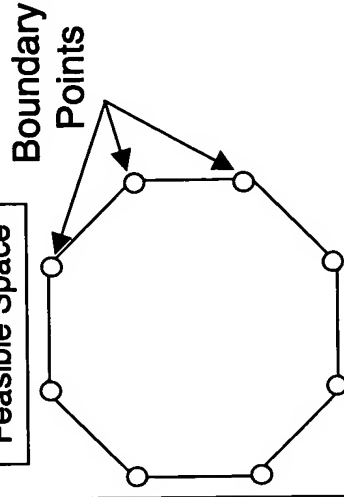
Multi-objective portfolio optimization problem is formulated as a problem with Multiple linear, nonlinear and nonlinear nonconvex objectives. However, the domain knowledge allows us to use strictly linear and convex constraints.

Linear Convex Feasible Space



Knowledge about geometry of feasible space (i.e. convexity), allowed us develop a feasible space boundary sampling algorithm (solutions archive generation). By knowing the boundary of the search space, we can exploit that knowledge to design efficient interior sampling methods.

Linear Convex Feasible Space



Convex crossover is a powerful interior sampling method, which is guaranteed to produce feasible offspring solutions. Given parents  $P_1$ ,  $P_2$ , it creates offspring  $O_1 = \lambda P_1 + (1 - \lambda)P_2$ ,  $O_2 = (1 - \lambda)P_1 + \lambda P_2$ . An offspring  $O_k$  and  $P_k$  can crossed over to produce more diverse offspring.

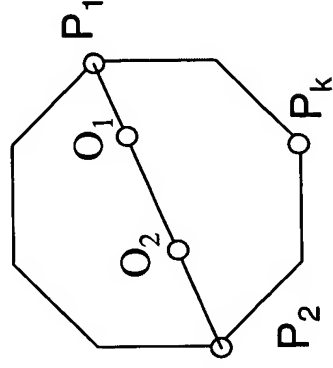


Figure  
36